Low frequency model of stacked film capacitors inductance

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Abstract: Polypropylene metallized capacitors are of general use in power electronics because of their reliability, their self healing capabilities, and their low price. Though the behavior of metallized coiled capacitors has been discussed, no work was done about stacked and flattened metallized capacitors.

The purpose of this article is to propose a simple analytical low frequency model of stacked capacitors. We solve the equation of propagation of the magnetic potential vector (**A**) in dielectric, in order to calculate the stray inductance of the capacitor.

We propose an original method of resolution, based upon the finite element method, in order to present an analytical but approximate solution of our problem. Then, we give some experimental results proving that the physical knowledge of the parameters of the capacitor (width, height, and thickness), enables the calculation of the stray inductance.

Key-Words: Capacitor, Stray-inductance, Finite Element Method, Galerkine method, Power Electronics.

1 Introduction

Polypropylene metallized capacitors are of general use in power electronics because of their reliability, their self healing capabilities and their low price. Nevertheless, the generalized use of these components does not imply a good knowledge of their behavior.

Our purpose is to determine a simple model of the stray inductance from the knowledge of geometric parameters of the capacitors (width, height, and thickness). The study is done for stacked and flattened film capacitors, in low frequency. Then we can neglect the radiated electromagnetic fields and the consequences of the environment (electromagnetic disturbances) considering that the capacitors could be shielded if necessary.

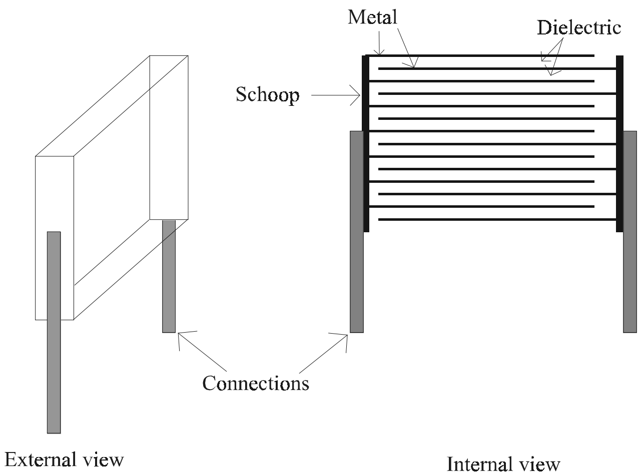

Fig. 1. External and internal views of stacked film technology capacitors.

2 Structure of a stacked film technology capacitor

Metallized stacked capacitors are made up of alternate dielectric and metal layers. The metal layer is typically composed of a few tenth of nanometers films of Zinc and/or Aluminium. The ends of the stack are sprayed with fine metal particles (Schoop process), which allows the connection to an external circuit.

3 Macroscopic model of capacitors

To calculate an analytical expression of the stray inductance, the internal current distribution must be known, as well as the internal magnetic field distribution. We know a propagation law in each medium [1, 2, 3], the continuity conditions between metal-dielectric and the boundary conditions, therefore we are able to solve the propagation equation (fig. 2).

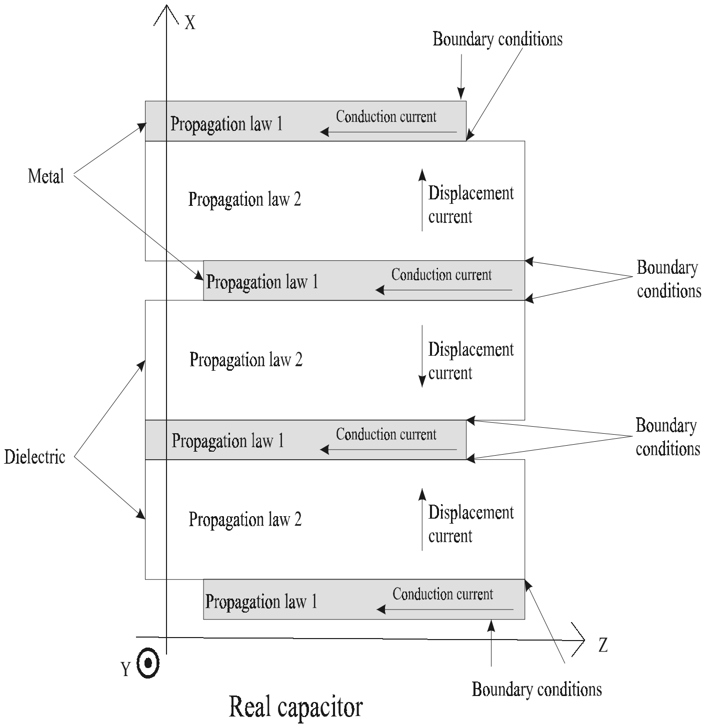

Fig. 2. Structure of the actual capacitor. The surfaces where boundary conditions must be taken into account are indicated.

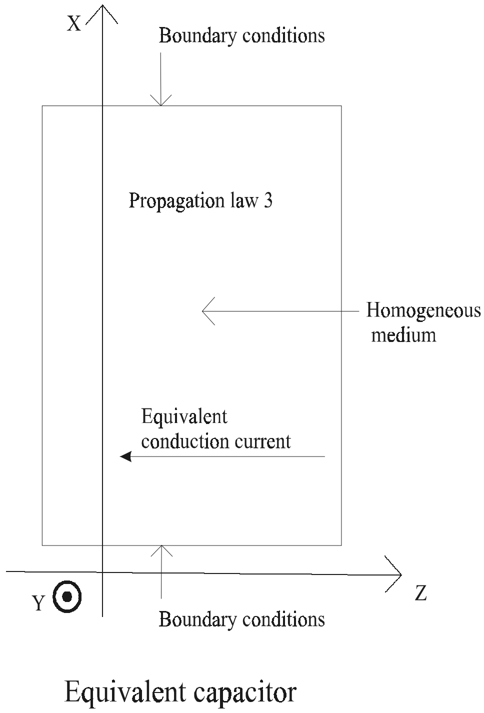

Fig. 3. Structure of the equivalent capacitor. The surfaces where boundary conditions must be taken into account are indicated.

From Maxwell equations

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$
$$\boldsymbol{B} = \mu.\boldsymbol{H}$$
$$\nabla \times \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{J} \qquad (1)$$

and Lorentz gauge

$$\nabla.\boldsymbol{A} + \varepsilon.\mu \frac{\partial \boldsymbol{V}}{\partial t} = 0 \qquad (2)$$

we obtain the propagation equation of potential vector **A** inside each layer of metal and dielectric (Fig. 2)

$$\Delta \boldsymbol{A} \;=\; \mu.\varepsilon \frac{\partial^2 \boldsymbol{A}}{\partial t^2} + \mu.\boldsymbol{J} \qquad (3)$$

At low frequency, equation (3) can be reduced

$$\Delta \boldsymbol{A} = \mu.\boldsymbol{J} \Leftrightarrow \mathfrak{L}(\boldsymbol{A}) + \boldsymbol{f}_v = 0$$
$$where \begin{cases} \mathfrak{L} = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2} \\ f_v = \mu.J \end{cases} \qquad (4)$$

Capacitors being made of about three thousand layers of metal and dielectric (fig. 2), we consider that a numerical determination of the current density would be better than an analytical one. Therefore, we must solve analytically the propagation law. In order to simplify the determination of the current and field distribution, we assume there is no change in capacitor behavior when an homogeneous material with appropriate characteristics takes place of the stack metal and dielectric (fig. 3 and 4). In a real capacitor, the current density is composed (fig. 2) by a displacement current in the dielectric and a conduction current in the metal. In the homogeneous capacitor, the current density is represented (fig. 3) by an equivalent current density. Because of the position of the metallic layers, the current density vector is parallel to the z axis. The vectorial propagation law (equation 4) gives three scalar equations, that can be reduced to one because the current density **J** and the potential vector **A** are collinear to the z axis. But **A** depends on the two other axis x and y. As a consequence, we can transform our 3D problem into a 2D problem and solve it on a surface parallel to the schoop (fig. 3). We assume that, on this surface, there is no field outside. The boundary conditions are (fig. 4):

$$A(x,y) = 0 \;\; on \;\; S \begin{cases} x = \pm \dfrac{X_0}{2} \\ y = \pm \dfrac{Y_0}{2} \end{cases} \qquad (5)$$

Since, considering that the capacitance of capacitors a and b must be equal (fig. 4), we have:

$$\left.\begin{aligned} C_{(a)} &= n.\varepsilon_0.\varepsilon_{r_1}.\frac{h.w}{e} \\ C_{(b)} &= \varepsilon_0.\varepsilon_{r_2}.\frac{w.l}{h} \end{aligned}\right\} \rightleftharpoons C_{(a)}=C_{(b)} \tag{6}$$

where $C_{(a)}$, $C_{(b)}$ are the value of capacitor (a) and (b) respectively.
ϵ_0 : permittivity of the vide.
$\epsilon_{r1}, \epsilon_{r2}$: relative permittivity of capacitor (a) and (b).
n : number of layers in capacitor (a).
w, l : width, height of capacitor (a) and (b).
e : thickness of the dielectric.
h : thickness of the homogeneous compound.

All the parameters are known except the relative permittivity of capacitor (b). It can be obtained by:

$$\varepsilon_{r_2} = \frac{h}{\varepsilon_0.w.l.C_{(a)}} \tag{7}$$

which implies a change in the value of the relative permittivity (ϵ_r). The capacitance of the capacitor remains constant. For example, the variation of permittivity between a 10 µF capacitor (l = 1.32cm, w = 1cm, h = 1.52cm) and its equivalent compound is about 10^8.

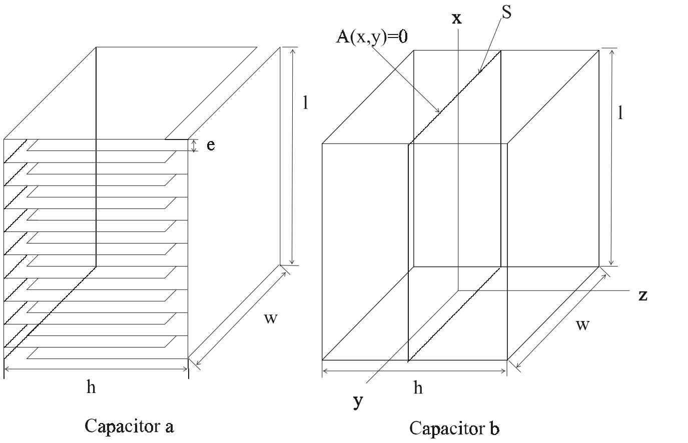

Fig. 4. Characteristic of the capacitors

4 Analytic resolution

We cannot solve equation (4) by a variable separation method because this equation cannot be written as $\mathscr{F}(A)=\mathfrak{L}(A)$ ($\mathscr{F}, \mathfrak{L}$ are linear operators). We are going to use finite element method [4], but instead of using n elements to represent the capacitor, we only use a single element that occupies the whole surface of the capacitor. The only restriction we have using this technic is that no brutal variation of current and field should occur inside the unique element. This condition can be assumed, provided that our model is restricted to low frequency. Therefore, in absence of the exact solution, the finite element method results in seeking the best approximate possible solution. To solve a differential equation such as,

$$\mathfrak{L}(u)+f_v=0 \tag{8}$$

an approach solution by the finite element method is

$$U(x_1,x_2,...)=<P_1(x_1,x_2,...),P_2(x_1,x_2,...),...>\begin{Bmatrix} a_1 \\ a_2 \\ ... \end{Bmatrix}$$
$$U(x_1,x_2,...)=<P>\{a\} \tag{9}$$

Where $<P_1(x_1,x_2,...),P_2(x_1,x_2,...),...>$ is a row vector composed by P_1, ..., P_n independent and linear functions, like polynomial or trigonometrical functions.

$\begin{Bmatrix} a_1 \\ a_2 \\ ... \end{Bmatrix}$ is a column vector formed by a_1, ..., a_n approximation parameters.

In order to simplify the analytical definition of arbitrary shapes and to clarify the solution of the equation (4), we have to introduce the reference element [5]. This reference element is a very simple shape (triangle, square), in a reference space, which can be transformed in a real element by a geometric transform $\mathfrak{I}$. The transformation depends on the shape and the position of the real element. Each transformation is selected as to present the following property: it is bijective in any point located on the reference element or its border. In other words, to each point of the reference space, corresponds one single point of real space, and inversely. For the plate capacitor, the reference form is a two unit wide square in an arbitrary unit.

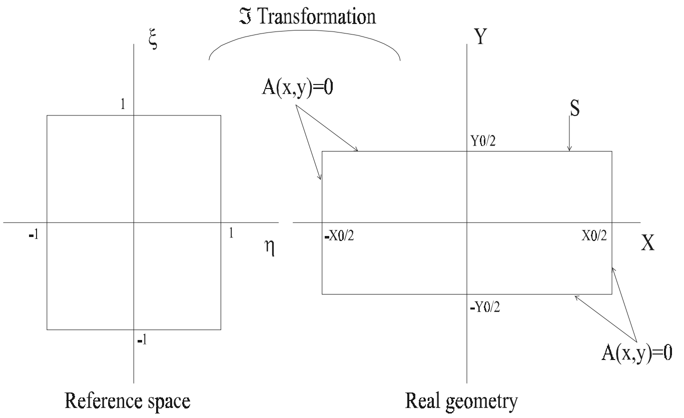

Fig. 5. Reference element for finite elements method

A solution, which satisfies the boundary and symmetry conditions (5) of equation (4), is :

$$U(x_1,x_2)=A(x,y)=P_1.a_1+P_2.a_2+P_3.a_3 \tag{10}$$

Where $P_1=(x^2-1).(y^2-1)$
$P_2=(x^2-1).(y^2-1).(x^2+y^2)$
$P_3=(x^2-1).(y^2-1).(x^2.y^2)$.

The three functions satisfy the boundary conditions and are linearly independent. The use of only three polynomials is a good compromise between accuracy and

speed computation. The approximation parameters a_1, a_2, a_3 can be calculated by different technics of approximation by weighted residues such as point collocation, subdomains collocation, Galerkine method, least error square method, Ritz method...
We used the Galerkine method, and least error square, because we find that the error is less important than in other methods and we present here only the Galerkine method.

4.1 Galerkine method

The method of the weighted residues [4] consists in seeking U functions which cancel the integral form

$$\int_v <\psi>(\mathcal{L}(u)+f_v)dv=0 \qquad (11)$$

for all weighted functions Ψ, also know as test functions, which belong to a space of functions E_Ψ. E_Ψ is a space vector of infinite dimension, composed by real valued functions. If the dimension of E_Ψ is finite, the solution U, which satisfies (8), is an approximate solution. The Galerkine method [4, 6, 7] uses as weighted functions the approximation function:

$$\int_v \{P\}(\mathcal{L}(<P>\{a\})+f_v)dv=0 \qquad (12)$$

It is routed in a procedure of minimization by orthogonalization which calls upon the properties of scalar product. We have a system of three equations with three unknown factors (a_1, a_2, a_3), the resolution of which permits us to find the optimal approximate solution.

$$\begin{cases} \int_v P_1\left(a_1.\dfrac{\partial^2 P_1}{\partial x^2}+a_2.\dfrac{\partial^2 P_2}{\partial y^2}+a_3.\dfrac{\partial^2 P_3}{\partial z}+f_v\right)dv=0 \\ \int_v P_2\left(a_1.\dfrac{\partial^2 P_1}{\partial x^2}+a_2.\dfrac{\partial^2 P_2}{\partial y^2}+a_3.\dfrac{\partial^2 P_3}{\partial z}+f_v\right)dv=0 \\ \int_v P_3\left(a_1.\dfrac{\partial^2 P_1}{\partial x^2}+a_2.\dfrac{\partial^2 P_2}{\partial y^2}+a_3.\dfrac{\partial^2 P_3}{\partial z}+f_v\right)dv=0 \end{cases} \qquad (13)$$

4.2 Calculating the stray inductance

Equation (13) gives us the potential vector **A** and, of course, the magnetic excitation **H** [8]. The electromagnetic energy W in the volume V of the capacitor is :

$$W = \frac{1}{2}\mu\iiint H^2 dv \qquad (14)$$

When a current flows in the capacitor, it creates a magnetic field which, in turns, generates a stray inductance.

The associated magnetic energy is:

$$W = \frac{1}{2}L.I^2 \qquad (15)$$

And

$$L = \frac{\mu\iiint_v H^2.dv}{I^2}$$

$$where \left.\begin{matrix}\boldsymbol{B} = \nabla\times\boldsymbol{A} \\ \boldsymbol{B} = \mu.\boldsymbol{H}\end{matrix}\right\} \quad L = \frac{\iiint_v(\nabla\times\boldsymbol{A})^2.dv}{\mu.I^2} \qquad (16)$$

Using equations (10), (13) and (16), the analytical expression of the stray inductance is [9]:

$$L=K.\frac{X_0.Y_0.Z_0}{X_0^2+Y_0^2} \qquad (17)$$

K is a constant, which is subordinated to the number of polynoms incoming from the method.

5 Experimentation

The validation of our model is based, in one hand, upon stacked capacitors from Siemens-Matsushita (fig. 7) and, in the other hand, upon data analysis concerning flat coil capacitors from SB Electronics (fig. 8).

Because no data is available for Siemens-Matsushita stacked capacitors, we recorded the curves of the impedance vs frequency on an HP 4194 A impedance analyzer. From the first resonance frequency of these curves $\omega = \frac{1}{\sqrt{L_s.C}}$, we deduce the stray inductance of the capacitor [8]

$$L_s=\frac{1}{(2.\pi.f)^2.C} \qquad (18)$$

In order to enforce the boundary conditions, we covered the capacitors with a copper ribbon when we carried out our measurements. As a consequence, the capacitor is shielded and no magnetic field is present outside the compound. Also, we take into account the stray inductance of the connections [10]:

$$L_e=\frac{\mu_0}{\pi}.\ln\left(\frac{2.D}{d}\right) \qquad (19)$$

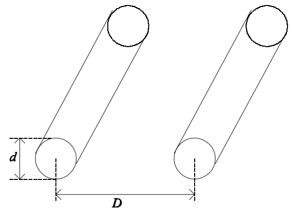

Fig.6. inductance of the connections

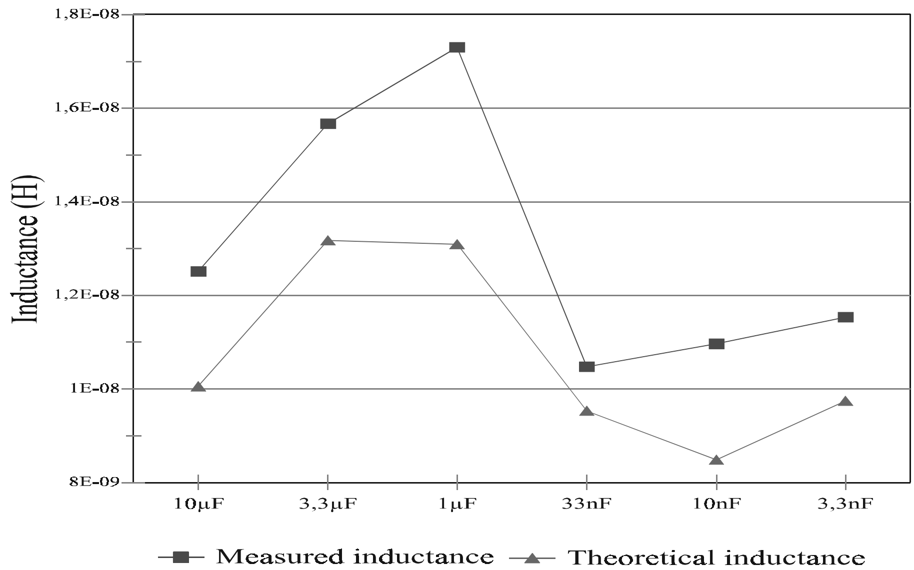

Fig. 7. Comparison between the value of the practical inductance (from HP4919A) and the calculated model (equations 17 and 19) for Siemens-Matsushita components.

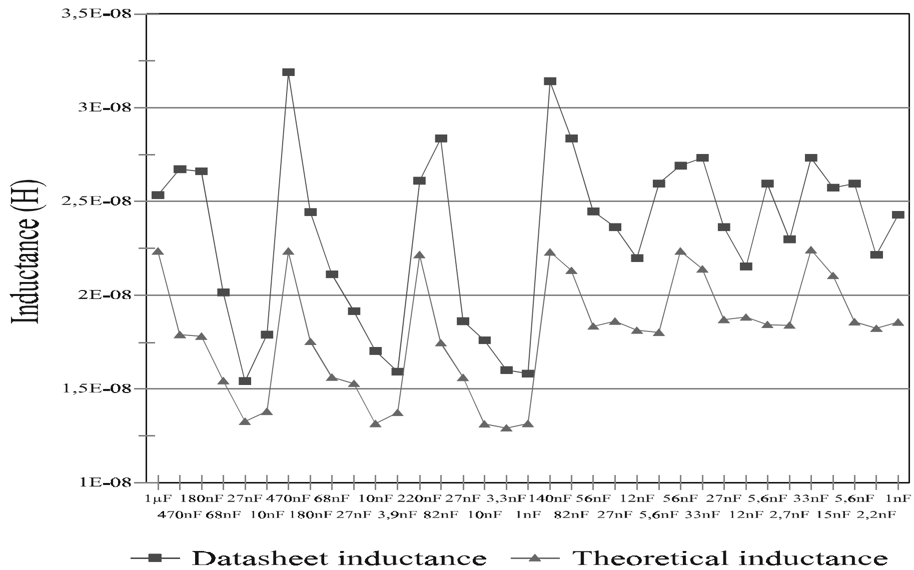

Fig. 8. Comparison between the value of the calculated from the SBE datasheets and the calculated model (equations 17 and 19).

The comparison between the model (equation 17 and 19) and the experimental results (equation 19) are given on fig.7. It can be seen that measured values of the stray inductance differ slightly from the expected ones. The difference is mainly due (fig. 7) to the difficulty to evaluate the stray-inductance of the external measuring circuit (soldering, connection cable, circuit).

SB Electronics technical information provides the curves of impedance vs frequency for capacitors with six different voltage ratings (200, 400, 600, 1000, 1200 and 2000V DC). Again, we deduce the stray inductance from the first resonance frequency (fig. 8: datasheet inductance). For calculation, flat coil capacitors, have been considered as equivalent to rectangular ones. Results are plotted on fig. 8.

The discrepancies between the two curves are primarily due to the fact that a flat coil capacitor is naturally more inductive than a stacked one, and the use of cartesian coordinates is perhaps not suited for these capacitors, elliptic coordinates, would be better.

6 Conclusion

From the magnetic field distribution, we have established the expression of the stray inductance of stacked film technology capacitor. By transformation of a discrete material (fig. 3: capacitor a) into an homogeneous system (fig. 3: capacitor b), we can simplify the expression of the problem. Then an analytical solution of the problem is possible. However, even with this simplification, analytical resolution of equation (6) in a cartesian system, is especially laborious and we use a numerical technic (finite element method) to calculate an analytical and approximated value of the stray inductance. We get it off from its usual work context, but in this case, it gives good results.

Extracting the internal stray inductance from fig. 7 (inductance without the linear inductance of the connections) shows that this structure has a stray inductance typically less than to 1nH. Now we have to calculate a high frequency model of the stacked capacitors, it will be our next job.

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